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Aircraft Time Constant and Flying "On the Step"

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Nomenclature

b = wingspan
C_{Dp} = profile drag coefficient
D = aircraft drag
e = Oswald efficiency factor
m = aircraft mass
P_s = engine shaft power
S = reference surface area
T = aircraft thrust
t = time
V = true airspeed
W = aircraft weight
ΔV = change in true airspeed from trim condition
η_p = propeller efficiency
ρ = air density
τ_j = jet aircraft time constant
τ_p = propeller aircraft time constant

Introduction

THERE is a belief among some pilots that it is possible to cruise an airplane at a higher speed than would normally be expected for a given power setting. This is known as flying "on the step" and is supposedly accomplished by climbing the airplane to an altitude several hundred feet higher than the intended cruise altitude. The pilot then dives the airplane at climb power back to the cruising altitude to attain a higher speed and then sets the engines at cruise power. If the airplane was on the step, it would presumably remain at this higher speed. However, in time the airplane would invariably fall "off the step" and return to its normal cruise speed.

This belief came about in World War II with the advent of large, low-drag, transport/bomber aircraft. The aircraft were often flown at long-range cruise airspeeds, and the pilots would try to increase this speed and stretch the range without raising fuel consumption with a higher power setting on the engines. The practice became so prevalent that the U.S. Army Air Corps published a Technical Order refuting the procedure and urging pilots to abandon this practice.[1] However, the belief in flying on the step continues today among some pilots, even though there is no theoretical basis for it.

Aircraft Response

To understand why the belief in flying on the step continues, it is necessary to look at how airplanes behave when disturbed from steady-state conditions and how this behavior can be described analytically. Flight tests have shown that when a propeller-driven airplane is given a change in power and constrained to fly in the horizontal plane, the airplane behaves as a first-order system, accelerating to a new steady-state trim velocity and approaching it asymptotically.[2] This first-order system is characterized by its time constant τ, the time the system takes to decrease to 37% of its initial disturbance. For cases of practical interest, the system will reach equilibrium in 4τ time periods.[3] Because the behavior of the airplane appears to be a first-order response, when the power and velocity are not in equilibrium, it is possible that the explanation to the concept of flying on the step lies in the components that make up the time constant. To derive an analytical expression for the time constant, a simplified analysis will be performed on an aircraft constrained to remain in level flight.

It is assumed that the thrust vector lies along the velocity vector, and that the airplane drag can be modeled by a parabolic drag polar. Initially, the airplane is in horizontal, steady-state flight, and so thrust equals drag and the equation of motion in the horizontal plane is

$$T_0 - \frac{1}{2} C_{Dp}\rho V_0^2 S - \frac{2(W/b)^2}{\rho\pi e V_0^2} = 0 \tag{1}$$

where the subscript 0 represents the initial steady-state condition. Now let the airplane be disturbed from its steady-state velocity V_0 to a new velocity V. Because thrust may also change with velocity, the new equation of motion for horizontal flight is written as

$$T_0 + \left(\frac{\partial T}{\partial V}\right)_0 \Delta V - \frac{1}{2} C_{Dp}\rho(V_0 + \Delta V)^2 S - \frac{2(W/b)^2}{\rho\pi e(V_0 + \Delta V)^2} = m\frac{d(\Delta V)}{dt} \tag{2}$$

where the change in thrust is represented as a first-order, Taylor-series expansion in velocity. Expanding the squared terms, neglecting ΔV^2 elements, and equating the initial thrust and drag yields

$$\left(\frac{\partial T}{\partial V}\right)_0 \Delta V - C_{Dp}\rho V_0 S\Delta V + \frac{4(W/b)^2\Delta V}{\rho\pi e V_0^3} = m\frac{d(\Delta V)}{dt} \tag{3}$$

This equation is a first-order linear-differential equation in ΔV, and the solution can be written as

$$\Delta V = \Delta V_i e^{-t/\tau} \tag{4}$$

where ΔV_i is the difference between the initial velocity V and the trim velocity V_0, and the time constant is expressed as

$$\tau = \frac{mV_0}{2\left[\frac{1}{2} C_{Dp}\rho V_0^2 S - \frac{2(W/b)^2}{\rho\pi e V_0^2}\right] - \left(\frac{\partial T}{\partial V}\right)_0 V_0} \tag{5}$$

Thus the response of the airplane to a velocity change is directly proportional to its initial momentum and inversely pro-

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portional to its drag characteristics and the relationship between the aircraft thrust and velocity.

Application to Propeller-Driven Aircraft

This expression can be simplified by evaluating the thrust term for propeller and jet aircraft separately. For propeller-driven aircraft, the relationship between thrust and velocity can be expressed as

$$T = \eta_p P_s / V \tag{6}$$

Taking the partial derivative of T with respect to V, evaluating it at the initial cruise condition, and multiplying by the initial velocity yields

$$\left(\frac{\partial T}{\partial V}\right)_0 V_0 = -\frac{\eta_p P_s}{V_0} \tag{7}$$

Thus, the term relating the product of change in thrust and velocity is simply the negative of the steady-state thrust. However, this value of thrust must equal the steady-state drag at V_0, and so the time constant τ_p for propeller-driven airplanes can be expressed as

$$\tau_p = \frac{\text{Momentum}}{3(\text{Profile Drag}) - \text{Induced Drag}} \tag{8}$$

where the momentum and the drag terms are evaluated at the steady-state condition. The denominator can be simplified by examining the expression for power required in level flight. Because the power required is the product of drag and velocity

$$\frac{\partial P}{\partial V} = \frac{3}{2} C_{Dp} \rho V^2 S - \frac{2(W/b)^2}{\rho \pi e V^2} \tag{9}$$

but the right-hand side of this equation is simply

$$\frac{\partial P}{\partial V} = 3(\text{Profile Drag}) - \text{Induced Drag} \tag{10}$$

Thus, the time constant for propeller aircraft can also be expressed as

$$\tau_p = \frac{\text{Momentum}}{\left(\dfrac{\partial P}{\partial V}\right)_0} \tag{11}$$

or aircraft momentum divided by the change in power-required with respect to velocity.

This expression for the time constant of a propeller-driven airplane explains why the concept of flying on the step came into being during World War II and has continued to this day. The airplanes developed during the second World War were much heavier and faster than the ones immediately preceding the war. Thus, they had much more momentum than those previously flown by the pilots. In addition, the aircraft were aerodynamically cleaner, which decreased the change in the power required with velocity. The operating conditions of these airplanes further increased the time constant. Because the bombers and transports were often flown at long-range cruise speeds, this meant that they were operating at a point on the power required vs velocity curve, where the slope of the curve was less. The combination of a heavy aircraft with low drag being flown at its long-range cruise speed all contributed to increasing the time constant of the airplane.

With these conditions it is clear how the concept of the step began. When the airplane was accelerated to a velocity slightly higher than its normal long-range cruise speed and the power set to the normal cruise condition, the airplane would appear to remain at that speed because of its long time constant. Seeing this, the pilot would assume that the aircraft was performing better than the performance charts indicated. However, eventually the airplane's speed would decay, thus giving the impression that the airplane had fallen off the step. In addition, if the pilot dived the airplane to initially get the airplane at the higher speed and the airplane remained in a very slight descent, the airplane would appear to maintain that speed until the altitude loss became apparent. The pilot would then correct that and the airplane would again be off the step.

To understand the length of time involved, a sample calculation was performed using a Douglas Aircraft DC-7C, the last of the piston engine, propeller-driven airliners. The long-range cruise speed and altitude for the DC-7C is given as 238 kn at 15,000 ft.[4] A drag buildup was performed on the airplane, resulting in an estimate of the parasite drag coefficient and wing efficiency of $C_{Dp} = 0.020$ and $e = 0.76$, respectively.[5] At the long-range cruise speed and a weight of 135,000 lb, the time constant τ_p is 211 s. Thus, it would take $>3\frac{1}{2}$ min for the increase in airspeed to decrease to 37% of initial value and approximately 14 min for the increase to disappear. It is easy to understand why pilots flying at these velocities would think they were flying on the step.

Application to Jet Aircraft

A similar analysis can be conducted for jet aircraft. For a first-order approximation, the thrust of a jet engine is essentially constant with velocity, and so the change in thrust with respect to velocity is equal to zero. The time constant τ_j for jet aircraft is then expressed as

$$\tau_j = \frac{\text{Momentum}}{2(\text{Profile Drag} - \text{Induced Drag})} \tag{12}$$

The denominator can be expressed in terms of the change in drag with respect to velocity, similar to the case of the propeller-driven aircraft. Taking the partial derivative of drag with respect to velocity, then multiplying each side by velocity, and evaluating the expression at the trim velocity yields

$$\left(\frac{\partial D}{\partial V}\right)_0 V_0 = 2\left[\frac{1}{2} C_{Dp} \rho V_0^2 S - \frac{2(W/b)^2}{\rho \pi e V_0^2}\right] \tag{13}$$

or

$$\left(\frac{\partial D}{\partial V}\right)_0 V_0 = 2(\text{Profile Drag} - \text{Induced Drag}) \tag{14}$$

The time constant for the jet aircraft can then be expressed as

$$\tau_j = \frac{\text{Momentum}}{\left(\dfrac{\partial D}{\partial V}\right)_0 V_0} \tag{15}$$

Thus, there is a fundamental difference between the time constant of the jet airplane and the propeller-driven airplane. Both are directly proportional to the momentum, but the propeller-driven airplane depends on the change in power with velocity, whereas the jet depends upon the product of the change in drag with velocity and the velocity. If two aircraft are identical in weight, velocity, and drag characteristics, but one is propeller-driven and the other powered by jets, the jet aircraft will have the longer time constant. With the same mass and velocity, the numerator of each time constant equation will be equal, but the denominator in the time constant equation of the propeller-driven airplane will always be greater than the denominator of the jet-powered airplane's time constant equation. This implies that the idea of flying on the step should be more prevalent among jet pilots. However, the longer equilibrium time is not normally a factor in long-range cruise for jets. The reason is that compressibility, which has been ignored in this analysis, begins to affect the drag of the airplane. Most modern jets cruise at fairly high subsonic Mach numbers. For example, the McDonnell-Douglas DC-10-10 long-range cruise speed varies, depending upon the weight,[6] from $M = 0.75$ to 0.83. If the pilot of a jet flying at a high subsonic speed dives the aircraft to accelerate to a higher speed with a fixed thrust, the drag coefficient increases more than indicated by the parabolic drag equation resulting from compressibility effects. The derivative term $\partial D/\partial V$ is thus much larger, and this results in a much lower time constant for the jet as it returns to equilibrium. Even with the thrust remaining constant, the larger increase in drag because of compressibility will tend to restore the airplane to its trim condition faster than predicted by incompressible analysis.

Conclusions

Some members of the pilot community have long insisted that it was possible to fly an airplane on the step, whereas engineers have generally ignored or ridiculed these claims. This analysis has shown why pilots have believed in this phenomena. The combination of high aircraft weight, low drag, and long-range cruise speeds typically near the minimum power-required velocity created larger aircraft time constants than previously seen by the pilots. This led to the illusion that they were flying on the step.

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Significance of Wash-Out on the Flutter Characteristics of Composite Wings

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Introduction

THE interest aeroelasticians have shown in exploiting composite materials to enhance aeroelastic stability has grown considerably, as evident from the literature on flutter and divergence characteristics of composite wings.[1–6] A compromise has been identified when altering the ply angles to achieve simultaneously higher flutter and divergence speeds.[1–4] There is a widely held view, that unfortunately is not always true, that ply orientation in a composite wing that results in a wash-in effect, i.e., bend-up/twist-up, has a beneficial influence on flutter, whereas the same effect is detrimental to divergence, with the opposite conclusion applied to wash-out. [For examples, see Fig. 5 of Ref. 1, p. 12, and Fig. 10 of Ref. 2, Conclusion on page 154 (paragraph 4) of Ref. 3 and Figs. 6 and 7 of Ref. 4.] These observations are made without due recognition of the central role of the modal coupling in aeroelastic studies. As a consequence, more attention has been focused on the flutter characteristics of composite wings that exhibit wash-in behavior while making a compromise on the divergence speed. Thus, relatively less emphasis has been placed on the flutter characteristics of composite wings that exhibit wash-out behavior. This Note redresses the imbalance and investigates the flutter characteristics of composite wings that exhibit wash-out behavior. In particular, the circumstances when wash-out can be advantageous in raising the flutter speed are identified, apparently for the first time. This study is particularly relevant because it is well recognized that wash-out is always beneficial for raising the divergence speed of a composite wing, whereas the widely held view is that it has an adverse effect on flutter.[1–4]

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Method of Analysis

The method of analysis is similar to the one used by the present authors in Ref. 6. However, to make this Note self-contained, certain features of the method are briefly summarized as follows.

1) The rigidities EI (bending), GJ (torsional) and K (bending-torsion coupling) of a composite wing for various ply angles are computed using the theory of Weisshaar and Foist[7] [see their Eqs. (18–20)]. Variation of these rigidities with ply angle enables the nondimensional uncoupled frequency ratio of the wing (ω_h/ω_α) to vary, which is later used to show the variation of flutter speed. (Note that ω_h is the fundamental uncoupled bending natural frequency, whereas ω_α is the corresponding fundamental uncoupled torsional natural frequency of the wing.) The bending-torsion coupling parameter ψ is defined in the same way as in Ref. 7 to give $\psi = K/\sqrt{EIGJ}$, so that the range for ψ is $-1 < \psi < 1$.

2) Next, the natural frequencies and mode shapes are computed using the dynamic stiffness matrix method put forward by Banerjee and Williams.[8] The normal modes obtained from this analysis are later used in the flutter analysis.

3) The flutter speed (V_F) is computed using the in-house computer program CALFUN,[9] which uses normal modes and generalized coordinates together with strip-theory aerodynamics. The nondimensional flutter speed $V_F/b\omega_\alpha$ (where b is the semichord) is plotted against the static unbalance x_α (defined as the nondimensional distance between the elastic axis and mass axis expressed as a fractional semichord, x_α is negative when the mass axis is behind the elastic axis) for a range of frequency ratios (ω_h/ω_α), and coupling parameters (ψ).

4) Finally, results obtained from using positive ψ, i.e., wash-out in the notation used in this Note, are compared with those obtained from using negative ψ, i.e., wash-in.

Discussion

Figure 1 shows the variation of the nondimensional flutter speed, i.e., $V_F/b\omega_\alpha$ against x_α for three different positive values of ψ, i.e., $\psi = +0.2$, $+0.4$, and $+0.6$, which induce the desired wash-out effect. Several representative values of the frequency ratio ω_h/ω_α were used in obtaining the results as shown in the figure. For comparison purposes, results for negative values of ψ, i.e., for $\psi = -0.2$, -0.4, and -0.6, are shown in Fig. 2. (It should be noted that the values of x_α plotted in Figs. 1 and 2 are all negative so that the mass axis is behind the elastic axis, which is usually the case.) The density ratio $m/\pi\rho b^2$ and the